$$\int_0^1 xe^x dx = \left[xe^x\right]_0^1 - \int_0^1 e^x dx$$
$$= e - \left[e^x\right]_0^1 = e - (e-1) = e - e + 1 = 1$$

D'où $\boxed{\int_0^1 xe^x dx = 1}$

*/ $\int_1^e \frac{\ln x}{x^2} dx$

$$U(x) = \ln x \to U'(x) = \frac{1}{x}$$
$$V'(x) = \frac{1}{x^2} \to V(x) = -\frac{1}{x}$$

Donc :

$$\int_1^e \frac{\ln x}{x^2} dx = \left[-\frac{\ln x}{x}\right]_1^e - \int_1^e -\frac{1}{x^2} dx$$
$$= -\frac{1}{e} - \left[\frac{1}{x}\right]_1^e = -\frac{1}{e} - \left[\frac{1}{e} - 1\right]$$
$$= -\frac{1}{e} - \frac{1}{e} + 1 = -\frac{2}{e} + 1$$

D'où $\boxed{\int_1^e \frac{\ln x}{x^2} dx = 1 - \frac{2}{e}}$

*/ $\int_1^x \ln t dt = \left[t \ln t - t\right]_1^x$
$$= (x \ln x - x) - (0 - 1)$$
$$= x \ln x - x + 1$$
$$\boxed{\int_1^x \ln t dt = x \ln x - x + 1}$$

*/ $I = \int_0^\pi e^x \cos x dx$
$$U(x) = \cos x \to U'(x) = -\sin x$$
$$V'(x) = e^x \to V(x) = e^x$$
$$I = \left[e^x \cos x\right]_0^\pi + \int_0^\pi e^x \sin x dx$$
$$= -e^\pi - 1 + \int_0^\pi e^x \sin x dx$$

Posons : $J = \int_0^\pi e^x \sin x dx$

Intégrant par parties J :

$$U(x) = \sin x \to U'(x) = \cos x$$
$$V'(x) = e^x \to V(x) = e^x$$

D'où on a :

$$J = \left[\sin x e^x\right]_0^\pi - \int_0^\pi e^x \cos x dx = 0 - I \Rightarrow J = -I$$

On trouve :

$$I = -e^\pi - 1 - I \Rightarrow 2I = -e^\pi - 1$$
$$\Rightarrow I = -\frac{(e^\pi + 1)}{2}$$

Conclusion :

$$\boxed{I = -\left(\frac{1 + e^\pi}{2}\right)}$$

*/ $\int_1^e x(\ln x)^2 dx$

$$U(x) = (\ln x)^2 \to U'(x) = 2.\frac{1}{x}.\ln x$$
$$V'(x) = x \to V(x) = \frac{x^2}{2}$$

Donc :

$$\int_1^e x(\ln x)^2 dx = \left[\frac{x^2}{2}(\ln x)^2\right]_1^e - \int_1^e (x \ln x) dx$$
$$= \frac{e^2}{2} - \int_1^e (x \ln x) dx$$

Intégrant par parties $\int_1^e (x \ln x) dx$:

$$U(x) = \ln x \to U'(x) = \frac{1}{x}$$
$$V'(x) = x \to V(x) = \frac{x^2}{2}$$

Donc :